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A Method to Determine Thermal Properties of Grain from System Response Analysis of a Packed Bed[1]

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A method for determining the particle thermal conductivity and particle-to-fluid heat transfer coefficient simultaneously for spherical particles is described. Small perturbation system analysis is used to minimize mass transfer and preserve linearity of response for moist grain particles. Preliminary tests with acrylic beads and soybean seeds demonstrate the accuracy of the method.

KEY WORDS: grain; heat transfer coefficient; packed bed; thermal conductivity.

1. INTRODUCTION

The thermophysical properties of biological materials, and in particular agricultural seeds, are not well known. These properties are essential to mathematical modeling involved in the development of control systems for grain drying. The parameters of specific interest are particle thermal conductivity, the particle-to-fluid heat transfer coefficient, and the axial fluid thermal dispersion coefficient. These are difficult to determine for moist heterogeneous particles such as grain kernels. Additional properties that are required either are available or can be readily determined with standard methods.

Grain is normally dried by forcing heated air through a loosely packed bed of kernels in a dryer column or grain bin. Therefore parameter estimation as a packed-bed system identification problem is an ideal method of determining the values for those properties of interest. That

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approach has been used successfully for beds of glass beads [1–3] and for fertilizer granules, iron ore pellets, and soybean seeds [4]. Pertinent published studies fall into two categories: frequency response analysis of packed beds and pulse response methods.

Thermal parameter estimation for moist material is complicated by coupling between the heat and the mass transfer processes. A large temperature gradient causes concomitant moisture migration, resulting in a highly nonlinear thermal response. Pulse response identification methods cannot be applied to the problem at hand for that reason. Preliminary work with soybean seeds, reported by Otten [4], indicates that frequency response analysis of moist seeds is also unsatisfactory due to high attenuation of the necessarily low-amplitude test signal.

A different approach using a pseudorandom binary noise sequence (PRBNS) and the correlation identification technique is described in this paper. This technique derives an impulse response for the fluid phase of the system. The small air temperature perturbations of the PRBNS signal (2.0 K) minimize moisture movement so that the thermal response about a particular operating temperature is linear. It must be emphasized, however, that any conclusions drawn from that response apply only in the vicinity of the operating temperature employed. Therefore the nonlinear system may be analyzed by investigating a number of operating temperatures within the range of interest. Parameter estimates can then be obtained from the thermal impulse response curve at each temperature.

2. THEORETICAL DEVELOPMENT

2.1. PRBNS System Identification

An excellent discussion of random signal system identification is given by Davies [5]. PRBNS signals are deterministic approximations of white noise used for statistical system testing. The test signal exists in only two states, either $+a$ or $-a$, where a is the amplitude of the signal. The states can change only at discrete time intervals, Δt, and the changes follow a deterministic sequence. The signal is periodic, with a period of $N\Delta t$. Maximum length sequences of a length $N = 2^m - 1$ are most frequently used and can be generated by an m-stage shift register with "exclusive or" feedback between stages.

The most remarkable property of a PRBNS signal is that its autocorrelation function (ACF) is very close to that of an ideal impulse when N is large. Since the signal is periodic, a cross-correlation function (CCF) between the input and the output signals can be identified after a minimum of two sequences. Using convolution integral theory, the CCF for a discretely sampled system with a PRBNS input yields the time impulse response directly, as

$$h(n\,\Delta t)=\frac{N}{a^2\,\Delta t(N+1)}\,\phi_{zu}(n\,\Delta t)+\frac{a^2}{N}\sum_{j=0}^{Nn}W_j \tag{1}$$

where

$$\phi_{zu}(n\,\Delta t)=\frac{1}{N}\sum_{r=0}^{N-1}Z_r U_{r-n} \tag{2}$$

The second term in parentheses in Eq. (1) represents the steady-state gain or bias term. It can be measured experimentally and subsequently removed.

2.2. System Model

The dispersion-concentric (D-C) model, as described by Wakao and Kaguei [6], is routinely used to describe unsteady-state heat transfer in a packed bed of spherical particles. Assumptions for the model are that the fluid phase is in the dispersed plug-flow mode and that individual particles exhibit a radially symmetrical temperature gradient. For seeds such as soybeans, which are roughly spherical, these conditions are met during forced air-drying. The system of equations comprising the model is

$$\alpha_{\mathrm{ax}}\frac{\delta^2 T_{\mathrm{f}}}{\delta x^2}-U\frac{\delta T_{\mathrm{f}}}{\delta x}-\frac{\delta T_{\mathrm{f}}}{\delta t}-\frac{3k_{\mathrm{p}}(1-\varepsilon)}{R\rho_{\mathrm{f}}C_{\mathrm{f}}\varepsilon}\left.\frac{\delta T_{\mathrm{p}}}{\delta r}\right|_{r=R}=0 \tag{3}$$

$$\frac{\delta T_{\mathrm{p}}}{\delta t}=\frac{k_{\mathrm{p}}}{(\rho C)_{\mathrm{p}}}\frac{\delta^2 T_{\mathrm{p}}}{\delta r^2}+\frac{\delta T_{\mathrm{p}}}{\delta r} \tag{4}$$

$$\frac{\delta T_{\mathrm{p}}}{\delta r}=\frac{h_{\mathrm{c}}}{k_{\mathrm{p}}}(T_{\mathrm{f}}-T_{\mathrm{p}})\qquad \text{at}\quad r=R \tag{5}$$

where C is the specific heat capacity, h_c is the convective heat transfer coefficient, k is the thermal conductivity, R is the particle radius, r is the radial displacement in a particle, T is the temperature, U is the interstitial air velocity, x is the axial displacement in the bed, α is the thermal diffusivity, ε is the bed porosity, and ρ is the density. The subscripts f and ax denote the fluid phase for the stagnant case and for flow in the axial direction, respectively, and the subscript p denotes the particle or solid phase.

An analytical solution for Eqs. (3)–(5) is possible after appropriate transformation. In the Laplace domain the solution is a transfer function for the inlet and outlet temperature of fluid passing through a bed of a finite length L:

$$F(s) = \exp\left[\frac{LU}{2\alpha_{ax}}(1 - \sqrt{1+b})\right] \tag{6a}$$

where

$$b = \frac{4\alpha_{ax}}{U^2}s + \frac{3k_p(1-\varepsilon)}{R^2\rho_f C_f \varepsilon}\frac{1}{(k_p/h_c R) + [1/(\phi \coth \phi - 1)]} \tag{6b}$$

and L is the length of a finite bed section, s is the complex Laplace variable, and $\phi = \sqrt{s}/\sqrt{\alpha_p}$.

2.3. Parameter Estimation

Determination of estimates for the heat transfer coefficient, the particle thermal conductivity, and the fluid axial thermal diffusivity follows a comparison of an observed system response to a calculated response using Eqs. (6a) and (6b), with iterative substitution of parameter values. A downhill simplex method of multidimensional minimization similar to the "amoeba" algorithm of Press et al. [7] was used to arrive at the best parameter estimates. That scheme does not require derivatives of the objective function to be minimized. A Laplace transform of the experimental response data was performed for several values of the complex Laplace variable, s. The optimization routine sought a minimum of the root mean square error between the transformed observed data and the calculated response.

3. EXPERIMENTAL EQUIPMENT AND PROCEDURE

Figure 1 is a diagram of the packed-bed test section of the apparatus used to determine the experimental response data. The accuracy of the heat transfer parameters depends upon the agreement between the physical system and the assumptions and constraints of the D-C mathematical model. The tubular test section is 146 mm in diameter and is mounted in a 3500-mm length of PVC pipe of the same inside diameter to ensure fully developed flow at the bed inlet. The bed-to-particle diameter ratio is greater than 10:1 for any of the packings studied.

Ambient air is conditioned to a particular humidity level and heated with an electrical resistance heater to the temperature selected for the test conditions. This airstream is split between the inner duct containing the particle bed and an outer concentric jacket, in order to avoid radial heat

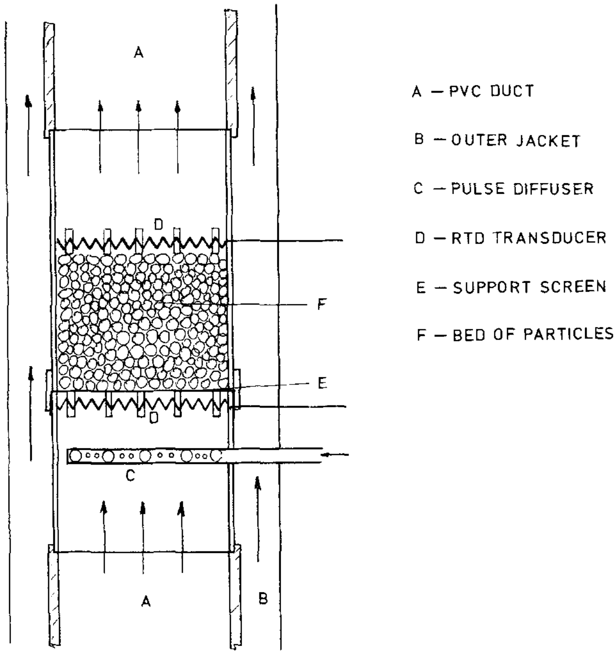

Fig. 1. Diagram of apparatus bed test section.

transfer due to a temperature gradient across the test section wall. The airflow balance is adjusted to give a bed interstitial velocity of 1.2 m/s, typical of commercial grain dryers.

The PRBNS pulse train is initiated by a personal computer that also monitors the air temperatures. A software-generated 7-bit shift register, with modulo two feedback from stages 4 and 7, determines the signal level. The register output logic level, either 0 or +5 VDC, opens one of a pair of solenoid valves which diverts a stream of compressed air to either a hot or a cold heat exchanger. The compressed air is introduced into the main airstream through a diffuser at the bed inlet, producing the series of small temperature perturbations. Air temperatures at the inlet and outlet planes of the bed are measured with a pair of resistance wire temperature detectors. These transducers were made from 4 m of 0.107-mm Balco wire to give a base resistance of 100.0 Ω at 293 K and a response time of about 0.2 s. The voltage drop across each transducer due to a constant current of 1.0 mA is monitored by the control computer. The PRBNS amplitude is 2.0 K and the sequence length is 127 bits, with a base pulse width of 15.0 s. At least four complete sequences of the PRBNS are run to minimize any effects of measurement noise in the system.

The D-C model describes the response of a finite section of an infinitely long bed. The experimental response, however, also contains the effects of the diffuser, the RTD transducers and the ends of the bed. A procedure using the responses for two different bed lengths removes these effects. The Laplace transform of the measured impulse response is a transfer function which is the product of individual transfer functions for the diffuser, the transducers, the end effects, and the bed effects. The quotient of response curves for two bed lengths, then, in the Laplace domain, is the true transfer function for a section equivalent to the difference in length between the two beds. In this study bed lengths of 76 and 38 mm were used.

Several preliminary tests have been conducted to demonstrate the validity of the method and confirm the operation of the apparatus. These were run with acrylic beads of 6.35-mm diameter and soybean seeds. The tests were conducted at the operating conditions of 313 K temperature and an absolute humidity of 0.007 kg/kg dry air. The seeds were held at those conditions in an environment chamber for 72 h prior to tests. Once the test section was packed and in place, the apparatus was allowed to attain thermal equilibrium. At that point the PRBNS signal was initiated and data were collected for several sequences. The test chamber was then repacked to a different length and the second test was run once equilibrium was reestablished. The duration of each test, excluding equilibration of the apparatus, was about 2 h.

4. RESULTS AND DISCUSSION

The impulse response curves derived for acrylic beads with two bed lengths at 313 K are presented in Fig. 2. A discrete Laplace transform of each data set was performed over a range of vaues for the complex variable s. The optimization routine was run to find the best values for the unknown variables that produced the closest fit between the corrected transfer function and the analytical solution given by Eqs. (6a) and (6b). It became apparent that the air temperature response was not sensitive to variations in the axial dispersion coefficient. At the flow rates used in heated-air grain drying, convection dominates. Shen et al. [2] investigated the effects of the dispersion coefficient at high flow rates and indicated that there was not a strong interaction with the other parameters for a Reynolds number of 229. Wakao and Kaguei [6] concluded that the axial fluid thermal dispersion coefficient can be adequately estimated from the expression:

$$\frac{\alpha_{ax}}{\alpha_f} = \frac{1}{\varepsilon}\frac{k_e^0}{k_f} + 0.5(\mathrm{Pr})(\mathrm{Re}) \tag{7}$$

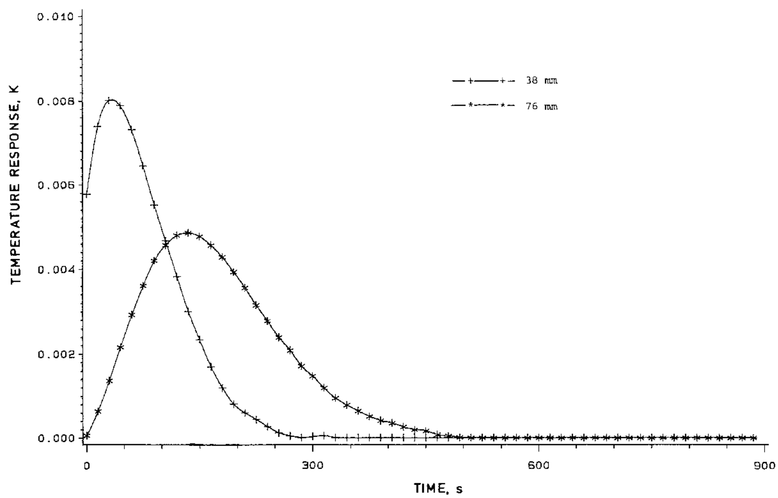

Fig. 2. Air temperature response curves for 76- and 38-mm beds of acrylic beads at 313 K.

where k_e^0 is the effective air thermal conductivity in the bed. Therefore in subsequent analyses the value of that term was calculated from Eq. (7) and only the heat transfer coefficient and the particle thermal conductivity were estimated from the experimental response. Values for the parameters determined are presented in Table I. The thermal conductivity of the acrylic beads was determined to be $0.193\ W \cdot m^{-1} \cdot K^{-1}$, a value within the range specified for the material [8]. The value of 24.1 for the Nusselt group was also very close to that predicted by an equation developed by Wakao et al. [8] from correlation of the results of many heat transfer studies. Similarly, the thermal conductivity for dry soybean seeds agreed closely with the value reported by Otten [4]. Convective heat transfer was about 20% higher for the soybeans than that predicted for smooth spheres of an equivalent size.

One difficulty with our method is in estimating the confidence level associated with the parameter estimates. The error term used to find the values in the s domain is calculated for the time integral of the responses for different weighting factors. Error analysis in the time domain is complicated by the cumulative effects of the transducers, the diffuser, and the bed ends contained in the observed responses. The analytical transfer function with the best parameter estimates was inverted using an IMSL library routine DINLAP to produce a calculated impulse response curve.

The calculated and observed responses for a 76-mm bed length of acrylic beads are compared in Fig. 3. A shift corresponding to a delay of

Table I. Thermal Parameters and Variables at 313 K

	Material	
	Acrylic beads[a]	Soybeans[b]
Mean diameter, mm	6.35	6.65[c]
Thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$		
Literature	0.188–0.201[d]	0.251[e]
Experimental	0.193	0.256
Particle Reynolds number	177	172
Nusselt group		
Literature[f]	25.5	23.8
Experimental	24.1	30.4

[a] Cords Canada Ltd., Toronto, Ontario.
[b] Maple Arrow variety, 4.5% moisture (w.b.).
[c] Mean diameter for volume-equivalent sphere.
[d] Source: Johnston Industrial Plastics, Toronto, Ontario.
[e] Source: Otten [4].
[f] Source: Wakao and Kaguei [6].

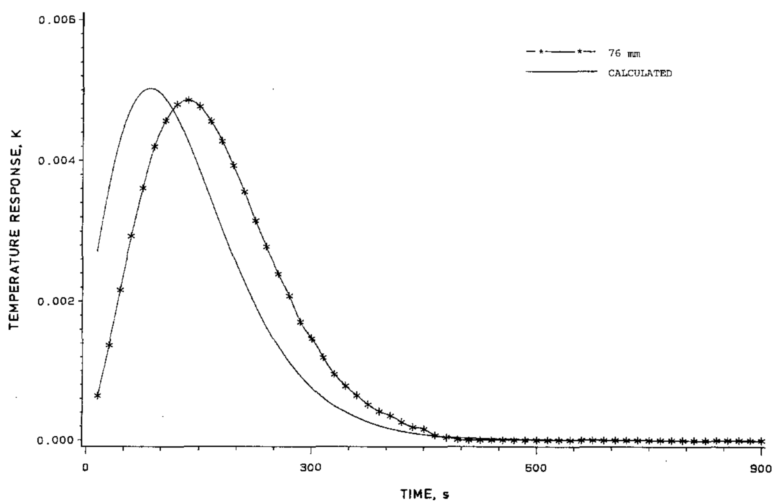

Fig. 3. Calculated and observed air temperature response curves for a 76-mm bed of acrylic beads at 313 K.

about 30 s appeared between each pair of response curves. This is due to the cumulative effects in the observed response noted earlier. The overall shape and magnitude of the two curves are virtually identical, indicating that the dynamics of the experimental response have been duplicated by the calculated response. On the basis of that, and the agreement between our results and known values of the parameters, we conclude that this method can produce reliable values for the thermal properties of interest.

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